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CONTROL OF THE VIBRATIONS OF A TETHERED SATELLITE SYSTEM†

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A method of damping structural vibrations using optimization techniques is presented and applied to a tethered satellite system. Such systems show large displacements and require active or passive damping mechanisms. A tethered satellite system is modelled by the method of multibody systems using symbolic equations of motion. Active damping is provided by an actuator between the main body and the tether. The control parameters are optimized. The energy decay of the system is used as the performance criterion. The complex dynamics of the motion of this system are demonstrated in simulations with different initial conditions including structural vibrations. It is shown that an optimization process enables the control parameters be improved with respect to the dissipation of energy of longitudinal structural vibrations. © 2001 Elsevier Science Ltd. All rights reserved.

A tethered satellite system consists of two or more satellites attached to each other by a long string. Such systems show high dynamic potential with various applications [1]. In this paper, the damping of structural vibrations during the tethered deployment of a reentry capsule from the International Space Station is considered [2].

The technique employed is based on the optimization of multibody systems [3, 4]. The modelling of mechanical structures as multibody systems is a well-established approach if large motions and small deformations occur [5–7]. For optimization purposes, the equations of motion of such models should be derived in symbolic form. The parameters can thereby be varied without a new derivation of these differential equations. This can be done automatically by computer codes, for example, by NEWEUL [8]. The optimization process itself requires algorithms for searching for a minimum, for integrating the equations of motion and possibly for a sensitivity analysis, which are included in the AIMS program [9].

The idea of using a system of two or more satellites connected by a long thin tether dates back several decades [10, 11]. The most realistic applications suggested are probably the creation of artificial gravity by two satellites circling each other, the gravitational stabilization of a two-body system, the launching of small satellites and the deorbiting of a reentry capsule. An overview over the suggested applications for tethered satellite systems is given in the book by Beletskii and Levin [1], which can be regarded as the standard work on space tether systems. This book treats the dynamical effects of massless and massive elastic tethers in various configurations and missions. There is also a literature on the deorbit manoeuvre, treated from the point of view of aeronautical and spacecraft engineering [2, 12].

As regards the dynamics of tethered satellite systems, a large number of papers have been published during the last few years. In particular, in [13–16] the control of tether vibrations is considered, and a suitable mechanical model is presented in [17]. The dynamical analysis of tethered satellite systems in [18] should also be mentioned.

1. FORMULATION OF THE PROBLEM

In this paper, the tethered deorbit of a reentry capsule without a rocket propulsion system is considered [12]. Either a static release (the left side of Fig. 1) or a dynamic one (the right side) is possible (S is the Space Station, T is the tether, P is the capsule, C is the tether cutoff point and ω_{orb} is the orbital angular velocity). In the first case, the capsule is slowly lowered radially to the Earth, resulting in a lower orbit velocity. In the second case, the tether is deployed more rapidly leading to a lateral offset due to the Coriolis force. When the tether deployment is stopped, the capsule swings back towards the local vertical, which corresponds to braking of the capsule. The advantage of a dynamical release is that the required tether length is only about half the length necessary for a static release.

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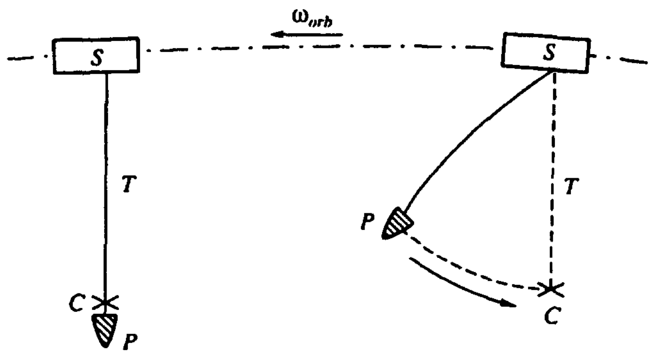

Fig. 1

In this paper, the back swing of the capsule during the dynamic release is considered and the damping of the resulting structural vibrations by active control is investigated. For this mission, the tether will typically have a length of 20 km with a diameter of only 0.5 mm. The mass of the return capsule is 170 kg, which corresponds to the prototype reentry capsule [2]. The parameters of the station are chosen to be those for the International Space Station, which is planned to fly with an overall mass of 415 t at an altitude of 400 km.

2. THE MECHANICAL MODEL

The tethered satellite system is modelled as a multibody system, as shown in Fig. 2. It consists of two rigid bodies in free space representing the station S and the return capsule with payload P. These two bodies are connected by a chain of n particles (point masses) which are interconnected by spring damper combinations representing an elastic tether. This model is similar to the bead model [17] but differs with regard to the two end bodies, which may move completely freely in space. The parameters of the particles and spring damper combinations are chosen such that they represent an equivalent element of the tether of length l:

$$m = \rho_T A_T l, \quad c = E_T A_T / l, \quad d = D_T / l$$

where ρ_T is the density, A_T is the cross-section, E_T is Young's modulus and D_T is the damping constant of the tether material. For a very long thin tether, usually Kevlar or Dyneema are considered as appropriate materials [19]. The length l of the tether elements depends on the discretization

$$l = L_T / n$$

where L_T is the overall length of the tether. For validation purposes $n = 50$ particles were used. In the optimization process $n = 20$ since the trajectories showed no significant differences.

The equations of motion are comparatively simple if all generalized coordinates are chosen as absolute coordinates given in the non-rotating Earth fixed system of coordinates

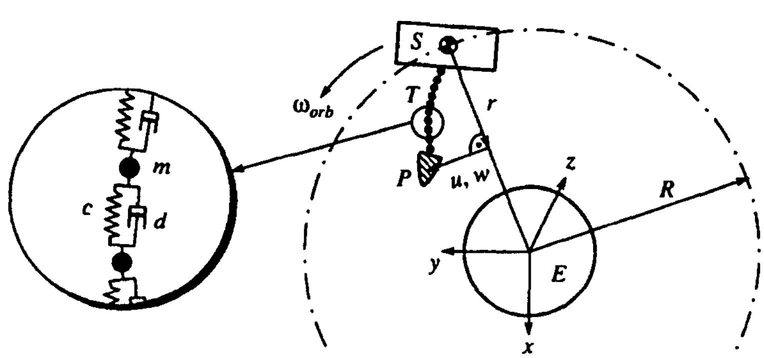

Fig. 2

$$\mathbf{y} = \| \mathbf{X}_S^T \ \mathbf{A}_S^T \ \mathbf{X}_1^T \ \mathbf{X}_2^T \ \dots \ \mathbf{X}_n^T \ \mathbf{X}_P^T \ \mathbf{A}_P^T \|^T$$

where $\mathbf{X}_i$ is the coordinate vector of ith body centre of mass and $\mathbf{A}_i$ is the vector of its attitude angles. The subscripts i denote the tether particles, $i = S$ for the Space Station and $i = P$ for the return capsule. The equations of motion then take the form

$$\dot{\mathbf{y}} = \mathbf{z}, \quad M\dot{\mathbf{z}} + \mathbf{k} = \mathbf{q} \tag{2.1}$$

where $\mathbf{z}$ is the time derivative vector of the vector $\mathbf{y}$, M is a positive definite block diagonal inertia matrix

$$M = \mathrm{diag}\| M_S \ m \ m \ \dots \ m \ M_P \|$$

$\mathbf{k}$ is a $(3n + 12)$ – dimensional vector whose non-zero elements correspond to gyroscopic terms quadratic in the velocities in the equations of motion of the end bodies, and $\mathbf{q}$ is the generalized forces vector, so that the force components for the ith particle take the form

$$\mathbf{F}_i = \| F_{xi} \ F_{yi} \ F_{zi} \|^T$$

It can be seen that the motions of the particles are coupled only via the generalized forces in the vector $\mathbf{q}$. For the x-direction these are

$$F_{xi} = F_{G_{xi}} + F_{C_{xi}} + F_{D_{xi}} - F_{C_{xi-1}} - F_{D_{xi-1}} \tag{2.2}$$

where $F_{G_{xi}}$ is the gravitational force on the ith particle, $F_{C_{xi}}$ and $F_{D_{xi}}$ are the spring and damper forces, respectively, between particles $i + 1$ and i and $F_{C_{xi-1}}$ and $F_{D_{xi-1}}$ are the spring and damper forces between particles i and $i - 1$. With m_E representing the mass of the Earth, γ the gravitational constant and l_0 the unstretched spring length, are obtain

$$F_{G_{xi}} = -m_E \gamma m \frac{x_i}{R_i^3}, \quad R_i = (x_i^2 + y_i^2 + z_i^2)^{1/2}$$

$$F_{C_{xi}} = c(l_i - l_0)\frac{x_{i+1} - x_i}{l_i}, \quad l_i = [(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2]^{1/2}$$

$$F_{D_{xi}} = d\frac{\dot{\mathbf{l}}_i^T \mathbf{l}_i}{l_i^2}(x_{i+1} - x_i), \quad \mathbf{l}_i = \begin{Vmatrix} x_{i+1} - x_i \\ y_{i+1} - y_i \\ z_{i+1} - z_i \end{Vmatrix}, \quad \dot{\mathbf{l}}_i = \frac{d\mathbf{l}_i}{dt}$$

It turns out that coupling between the directions occurs only via non-linear terms. Linearization about the quasi-static equilibrium position

$$z_i = z_{i0} + dz_i, \quad z_{i0} = 0, \quad \dot{z}_{i0} = 0, \quad \ddot{z}_{i0} = 0$$

leads to a decoupling of the in-plane and out-of-plane motions [16]. Since the coupling is an important effect when investigating the damping of a tethered satellite system, the complete non-linear equations are considered in this paper.

To reduce the vibrations by active damping control of the tether, a winch is employed. It is represented in the model by a force actuator at the space station S. Therefore, the actuator force F_{act} is added to the applied forces in the vector $\mathbf{q}$, e.g. the term $F_{\mathrm{act}}(x_S - x_1)/l_S$ is added to $F_{xi} (i = 1)$ in Eq (2.2).

As control laws for the actuator force F_{act} several linear relations can be found in literature [15]. For longitudinal control the following is recommended

$$F_{\mathrm{act}} = k_1 l + k_2 \dot{l} \tag{2.3}$$

where $l \equiv l_S$ is the distance between the station and the uppermost particle. For lateral control the following linear law can be used

$$F_{\mathrm{act}} = k_3 \theta + k_4 \dot{\theta}$$

where θ is the angle of inclination of the tether measured at the station. The non-linear relation

$$F_{act} = k_5\dot{\theta}^2$$

has also been proposed [13].

To reduce the influence of high-frequency oscillations on the controller a PT_1 filter is used between the sensor and the controller with a cutoff frequency of 1 s^{-1}. This frequency is significantly higher than the vibration frequency.

3. ANALYSIS OF THE MOTION OF THE TETHERED SATELLITE SYSTEM

The dynamics of the tethered satellite system were investigated numerically in two cases. The first case describes vibrations in the tether during station-keeping. The second case describes the motion during a three-dimensional transverse swing of the tether, where the effects of the non-linear coupling terms can be observed.

In both cases the station moves around the Earth as described in Section 1, but the orbit is not an ideal circle due to small perturbations of the initial conditions and structural vibrations.

Stationkeeping. In the stationkeeping operation the tether hangs straight down from the station with the payload pointing towards the centre of the Earth, while the complete system is moving in a near circular orbit. In order to compare the simulation results, the parameters are chosen as in [14].

$$L_T = 100\,\text{km},\quad \rho_T = 5.76\,\text{kg/km},\quad M_P = 500\,\text{kg},\ EA_T = 2.8\times10^5\,\text{N}$$

Additionally, a small internal damping $D = 2500$ N s, as in [19], is taken into account. Initially, the system is experiencing a large longitudinal vibration with no declinations from the vertical.

The results of the simulation are shown in Fig. 3, where r_P is the depth of the payload below the station, always pointing towards the centre of the Earth, and u_i is the transverse in-plane motion of the ith particle or the pay-load P, respectively, perpendicular to r. N on the lower scale is the number of orbits. It can

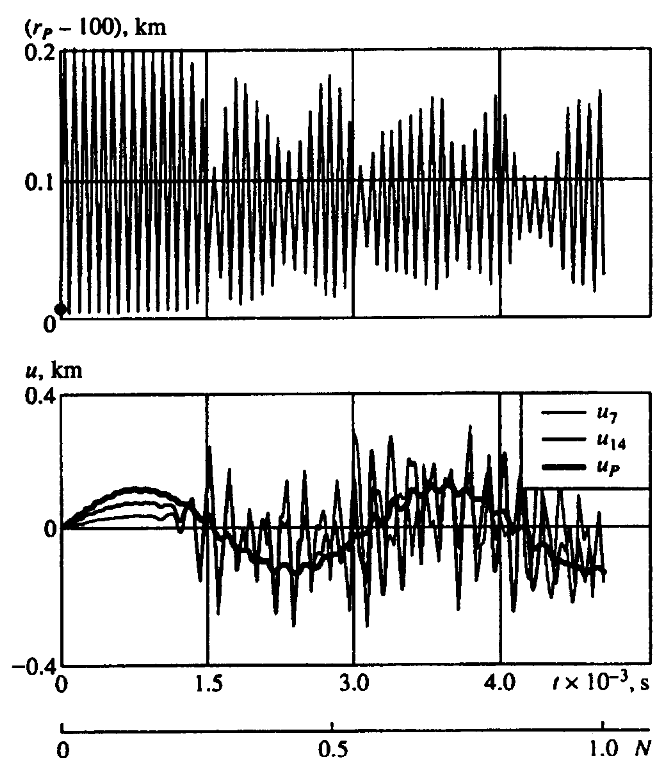

Fig. 3

be seen that the longitudinal vibration also leads to a transverse vibration of the tether and a small vibrational motion of the payload due to the Coriolis force. Obviously the energy initially stored in the longitudinal vibration is being transferred to transverse in-plane motions.

The normalized frequencies $\omega_0 = \omega/\omega_{orb}$ of the lower modes resulting from the simulation are listed below. The corresponding values obtained with the linear model [14] are shown in brackets for comparison. The librational frequency is equal to 1.758 (1.732), the first longitudinal frequency is 55.7 (54.3) and the first transverse frequency is 57.7 (59.6).

The frequencies agree quite well. The small differences can be explained by non-linear effects. For a simplified and completely linearized model with small displacements, the vibrational frequency of $\sqrt{3}\omega_{orb}$ can be calculated analytically [20]. The difference of 1.5% between the analytically calculated vibrational frequency and that obtained by simulation can be explained by the non-linear vibrational motion and the slightly elliptic orbit of the station.

A three-dimensional swing. The second case shows a vibrational swing of the tethered satellite. The tethered satellite has initially a large transverse deviation from the quasistatic equilibrium position in both the in-plane and out-of-plane direction. The parameters are chosen for the mission also described in Section 1 as

$$L_T = 20\,\text{km},\quad \rho_T = 0.3\,\text{kg/km},\quad M_P = 170\,\text{kg},\; EA_T = 3.15\times10^4\,\text{N},\quad D = 2500\,\text{N s}$$

The results of the simulation are shown in Fig. 4 where r_P is again the depth of the payload below

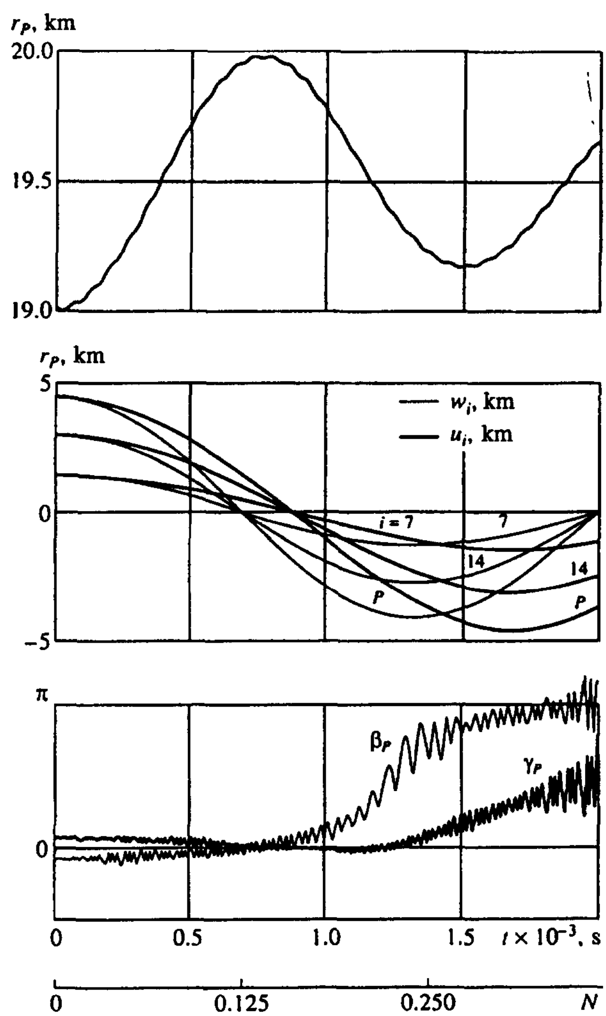

Fig. 4

the station, u_i is the transverse in-plane motion and w_i is the transverse out-of-plane motion. The angles of the payload β_P and γ_P relative to the y and z axes are measured in the body fixed system of coordinates. In addition to the motion of the payload, the transverse motion of the 7th and 14th particle are shown. Small longitudinal vibrations have no appreciable influence on the large-scale transverse motion. However the longitudinal motion of the tether excites a tumbling motion of the payload via non-linear coupling. When the vibrational swing completes a half period, invertion of the payload occurs. This is clearly a very undesirable effect, especially, when the payload is to be cut off and to enter the atmosphere along a specified path. Therefore an additional damping mechanism to control the payload motion is necessary. This simulation shows that the payload cannot in general be modelled as a point mass. It is necessary to use the rigid body model with rotational degrees of freedom.

4. OPTIMIZATION

To reduce the vibrations of the tether active damping by the tether winch is used. The controller gains and the structural parameters are adapted to each other by an optimization process. Since the optimization requires time consuming integrations of the equations of motion for each set of design variables, a deterministic algorithm must be chosen that requires a relatively small number of iteration steps. In this paper the algorithm from [21] is used, which is based on the method of sequential quadratic programming (SQP) [22]. Apart from function evaluations the algorithm requires the calculation of sensitivities which are computed by Automatic Differentiation [23].

As a reference for evaluating the dynamical behaviour of the tethered satellite system, the back-swing of the payload, as described in Section 1, is considered using the above parameters. The initial conditions for the system of differential equations (2.1) follow from a tether that is hanging in a straight line with a large lateral in-plane direction. Additionally, a small longitudinal direction is assumed, so that longitudinal and lateral oscillations occur.

If structural parameters such as the tether stiffness are varied during the optimization process, it is recommended that the initial conditions are defined in such a way that the initial energy is independent of these design variables, e.g. the energy stored in the stiffness is constant. This is of particular importance when the energy of the vibrations is taken as an optimization criterion. Other possible criteria are the displacements of the payload or the tether, i.e. displacements of some particles.

In this paper optimization with respect to the longitudinal tether vibrations is considered using the energy criterion described above

Optimization with Respect to Longitudinal Vibrations. To reduce the longitudinal vibrations the control law (2.3) for the actuator is simply chosen in the form

$$F_{act} = k_1 l + k_2 \dot{l}$$

where the gains k_1 and k_2 are design variables. The optimization criterion is the energy criterion

$$E = \int_0^T \left(\frac{1}{2} M_P (\mathbf{v}_P - \mathbf{v}_{ref})^2 + \frac{1}{2} c \Delta l_P^2\right) dt, \quad v_{ref} = \omega_{orb} R_P$$

where R_P is the distance of the payload from the centre of the Earth and $\Delta l_P = \Delta l_P(t)$ is the overall stretch of the tether.

The results are given below

Parameter	k_1/c_1	k_2/d_1
Initial value	0	10
Optimized value	−0.275	101.08
Limits of variation	−0.5 ... 10.0	0 ... 1000

and shown in Fig. 5. The thick curve in the left part shows the low-frequency motion; the thin curves in both graphs correspond to the initial values of the parameters, while the curves of normal thickness represent the optimized parameters. W_T is the potential energy in a stretched tether.

The optimized parameters lead to smaller amplitudes of the longitudinal vibrations while the large-scale motion is not affected. This is shown in Fig. 5 for a depth r_P around the tether cutoff point at

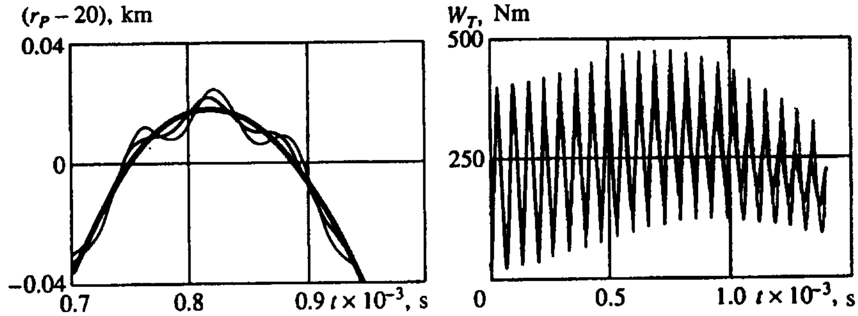

Fig. 5

about 800 s. In this graph the optimized dynamics are shown with respect to low frequency motion without structural vibrations. It can further be seen that the energy in the stiffness is dissipated more rapidly than with the initial parameters and, of course, also more rapidly than without active damping. Obviously, active control of the tether winch is more suitable for the purpose of providing additional damping to the tethered satellite system and stabilizing of motion in this way if appropriate parameters are chosen.

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